1. The following are unrelated: (15 pts)

   (a) Rewrite each of the following without the absolute value symbol:

   i. $|2 \quad - 6|$

   Solution:

   Since $\quad > 3$, $2 \quad > 6$, $2 \quad - 6 > 0$ so $|2 \quad - 6| = \boxed{2 \quad - 6}$

   ii. $|\sqrt{2} - 2|$

   Solution:

   Since $\sqrt{2} < 2$, $\sqrt{2} - 2 < 0$ hence $|\sqrt{2} - 2| = \boxed{-(\sqrt{2} - 2)}$ or 2 2

(d) Add and simplify: $\frac{2}{\frac{9}{7}} + \frac{5}{12} + 7^0$

Solution:

$$\frac{2}{\frac{9}{7}} + \frac{5}{12} + 7^0 = \frac{14}{9} + \frac{5}{12} + 1 \tag{4}$$
$$= \frac{56}{36} + \frac{15}{36} + \frac{36}{36} \tag{5}$$
$$= \boxed{\frac{107}{36}} \tag{6}$$

(e) Simplify: $\frac{|-7-3| + |2|}{2|-4|}$

Solution:

$$\frac{|-7-3| + |2|}{2|-4|} = \frac{10+2}{8} \tag{7}$$
$$= \frac{12}{8} \tag{8}$$
$$= \boxed{\frac{3}{2}} \tag{9}$$

2. The following are unrelated. Leave your answers without negative exponents. (20 pts)

(a) $(-5b^3)^2 7a^{-3}a^6$

Solution:

$$(-5b^3)^2 7a^{-3}a^6 = (-5)^2(b^3)^2 7a^3 \tag{10}$$
$$= 25b^6 7a^3 \tag{11}$$
$$= \boxed{175b^6a^3} \tag{12}$$

(b) Simplify: $\frac{\sqrt{32x^2}}{\sqrt{2}\sqrt{16}}$

Solution:

$$\frac{\sqrt{32x^2}}{\sqrt{2}\sqrt{16}} = \frac{|x|\sqrt{2\cdot 16}}{\sqrt{2\cdot 4}} \tag{13}$$
$$= \frac{|x|4\sqrt{2}}{2\sqrt{2}} \tag{14}$$
$$= \boxed{2|x|} \tag{15}$$

(c) Simplify: $\frac{2(x^{-2}y^3)^3}{8x^{-3}y^{-1/3}}$

Solution:

$$\frac{2(x^{-2}y^3)^3}{8x^{-3}y^{-1/3}} = \frac{2x^{-6}y^9}{8x^{-3}y^{-1/3}} \tag{16}$$
$$= \frac{x^{-3}y^{\frac{27}{3}}}{4y^{\frac{1}{3}}} \tag{17}$$
$$= \boxed{\frac{y^{\frac{28}{3}}}{4x^3}} \tag{18}$$

(d) Multiply to rewrite as a polynomial: $\left(\sqrt{x-1}+3\right)\left(\sqrt{x-1}-3\right)$

Solution:

$$\left(\sqrt{x-1}+3\right)\left(\sqrt{x-1}-3\right) = \left(\sqrt{x-1}\right)^2 - 3^2 \quad (19)$$

$$= x - 1 - 9 \quad (20)$$

$$= \boxed{x - 10}$$

(b) Simplify the compound fraction: $\dfrac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1}$

Solution:

$$\frac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1} = \frac{\frac{3-x}{x^2}}{\frac{9-x^2}{x^2}} \tag{27}$$

$$= \frac{3 - x}{9 - x^2} \tag{28}$$

$$= \frac{3 - x}{(3 - x)(3 + x)} \tag{29}$$

$$= \boxed{\frac{1}{3 + x}} \tag{30}$$

(c) Factor by grouping: $9x^3 - 18x^2 - 4x + 8$

Solution:

$$9x^3 - 18x^2 - 4x + 8 = 9x^2(x - 2) - 4(x - 2) \tag{31}$$

$$= (x - 2)(9x^2 - 4) \tag{32}$$

$$= (x - 2)\ \ (3x)^2 - 2^2 \tag{33}$$

$$= \boxed{(x - 2)(3x + 2)(3x - 2)} \tag{34}$$

5. Is $x = -2$ a solution of the inequality $x^3 - 2x < 2$

7. Solve each of the following equations. If there are no solutions write NO SOLUTIONS: (10 pts)

(a) $\sqrt{8 - y} + 2 = y - 4$

Solution:

$$\sqrt{8 - y} + 2 = y - 4 \quad (40)$$
$$\sqrt{8 - y} = y - 6 \quad (41)$$
$$8 - y = y^2 - 12y + 36 \quad (42)$$
$$y^2 - 11y + 28 = 0 \quad (43)$$
$$(y - 7)(y - 4) = 0 \quad (44)$$
$$y = 4; 7 \quad (45)$$

Plugging into the original equation, we find $y = 4$ to be extraneous. Hence $\boxed{y = 7}$

(b) Solve for h: $P = A + hdg$

Solution:

$$P = A + hdg \quad (46)$$
$$hdg = P - A \quad (47)$$
$$h = \boxed{\frac{P - A}{dg}} \quad (48)$$

8. Solve the following inequalities. Justify your answers by using a number line or sign chart if needed. Answers without full justification will not receive full credit. Express all answers in interval notation. (8 pts)

(a) $-3x + 1 < 6$

Solution:

$$-3x + 1 < 6 \quad (49)$$
$$-3x < 5 \quad (50)$$
$$x > -\frac{5}{3} \quad (51)$$

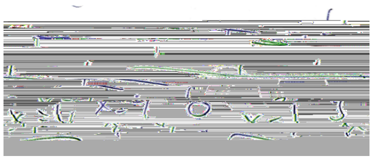
Hence the interval of solution is $\boxed{-\frac{5}{3}; 1}$

(b) $x^3 - 3x^2 \geq 0$

Solution:
We start by factoring the left hand side, and then make use of a number line/sign chart to choose the relevant interval of solution

$$x^3 - 3x^2 \geq 0 \tag{52}$$
$$x^2(x - 3) \geq 0 \tag{53}$$

Setting the left side equal to zero we get two values that make the left side zero: $x = 0$ and $x = 3$. Placing these on a number line and picking test values we obtain the following chart



Notice that $x = 0$ is a solution. Hence the solution is $\boxed{0 \cup [3, \infty)}$.

9. Find all the solutions to the following equation, including the complex solutions (Hint: factoring will be important) $z^3 = 1$. (5 pts)

Solution:

$$z^3 - 1 = 0 \tag{54}$$
$$(z - 1)(z^2 + z + 1) = 0 \tag{55}$$

From which we conclude that $\boxed{z = 1}$ or

$$z = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} \tag{56}$$
$$= \frac{-1 \pm \sqrt{-3}}{2} \tag{57}$$
$$= \boxed{\frac{-1 \pm \sqrt{3}i}{2}} \tag{58}$$