Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use one page of notes. You cannot collaborate on the

2. (26 points) Consider the plane given by

$$3x - 2y + z = 2$$

and the line given by the symmetric equations

$$\frac{x+1}{2} = y = 2 - z.$$

(a) (6 points) Find the point where the line intersects the plane.

Solution: We substitue the parameterized equations for this line:

$x = 2t - 1$
$y = t$
$z = 2 - t$

into the equation for the pane and solve for t:

$3(2t - 1) - 2(t) + (2 - t) = 2$
$6t - 3 - 2t + 2 - t = 2$
$t = 1$

Substituting this value of t back into our parameterized equations gives

$x = 2 - 1 = 1$
$y = 1$
$z = 2 - 1 = 1$

So the point is $(1; 1; 1)$.

(b) (6 points) Find the angle between the line and plane's normal vector.

Solution: We can read o the line's direction vector, $\mathbf{v} = < 2; 1; -1 >$, and the normal vector of the plane, $\mathbf{n} = < 3; -2; 1 >$, directly from their equations. We then nd that

$\mathbf{v} \cdot$

so $(3, 2, 0)$ is on the line.

(d) (8 points) Determine how far the point A is from the plane.

Solution: The point $(1, 1, 1)$ is on the plane, so we find the vector from this point to $(3, 2, 0)$. The absolute value of the scalar component of this vector along the plane's normal vector is the distance :

$\mathbf{u} = < 2, 1, -1 >$

$$d = \frac{|\mathbf{n} \cdot \mathbf{u}|}{|\mathbf{n}|} = \frac{3}{\sqrt{14}}$$

3. (20 points) A quadric surface is defined by the equation

$$2x^2 - 4x + y^2 - z^2 - 2z = 0$$

(a) (8 points) Classify the quadric surface and state its orientation.

Solution: We must complete the squares for x and z to get the quadric in a standard form:

$$2(x^2 - 2x) + y^2 - (z^2 + 2z) = 0$$
$$2(x^2 - 2x + 1) + y^2 - (z^2 + 2z + 1) = 2 - 1$$
$$2(x - 1)^2 + y^2 - (z + 1)^2 = 1$$

We see this is a hyperboloid of one sheet oriented parallel to the z-axis.

(b) (6 points) Find the center of the surface.

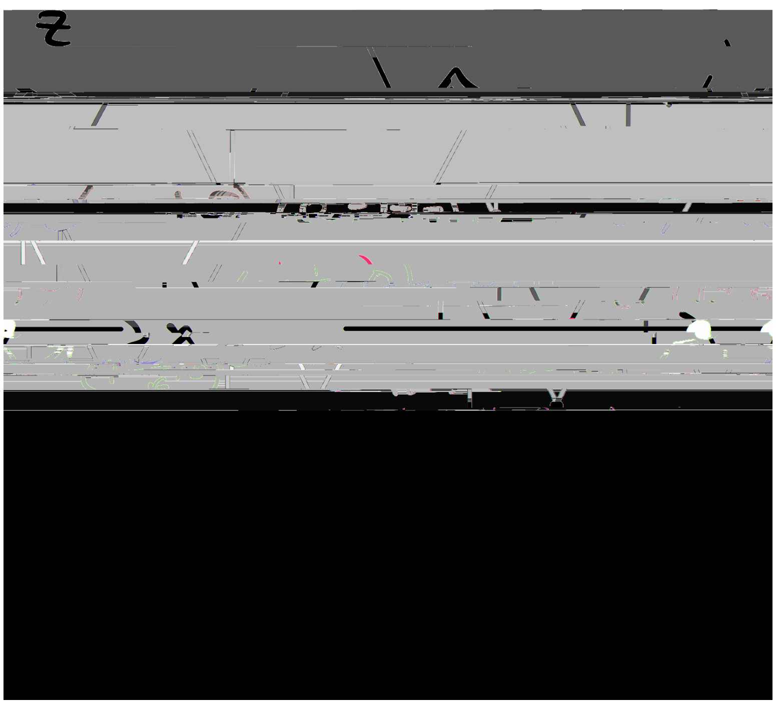
Solution: Reading offsets from the standard form above, we see that the center is at $(1, 0, -1)$

(c) (6 points) Sketch the trace for $y = 1$. Label all x and z intercepts.

Solution: When $y = 1$ we have

$$2(x - 1)^2 = (z + 1)^2$$
$$z + 1 = \pm\sqrt{2}(x - 1)$$
$$z - (-1) = \pm\sqrt{2}(x - 1)$$

So we have two lines which intersect at $(1, -1)$ in the xz plane with slopes $\pm\sqrt{2}$:



4. (28 points) An ion moving in a magnetic eld starts at the origin and follows a path given by the position vector

$$\mathbf{r}(t) = \quad \frac{t}{2} + \sin(t); \cos(t) - 1; \frac{3t}{8}$$

(a) (8 points) After the ion has moved 3 in the z direction, the ion collides with a neutral atom. Where does this collision occur? How far from the origin does the collision occur?

Solution: We know that the ion's position vector has a z component of 3 , so we use this to nd the time t of the collision and substitute it into the other components to nd the position:

$$\frac{3t}{8} = 3$$

$$t = 8$$

$$\mathbf{r}(8\ ) = \quad 8$$

(d) (6 points) In the collision, the ion becomes a neutral atom and continues traveling in a straight line afterwards. Find an equation for this line.

Solution: The line contains the point $\mathbf{r}(8\ )$ and has a direction vector $\mathbf{r}'(8\ )$:

$\mathbf{r}'(8\ ) = \mathbf{v}(8\ ) = \frac{3}{2}$ ⟨==) 0 Tf 6.1778 81m9.9626 Tf 6.175.6F8tTf 0 -25.903 Td 26 6 Tf 6.036 0 T.a